DESY 09-125 SFB/CPP-09-77

# 2- and 3-Loop Heavy Flavor Corrections to Transversity

Johannes Blümlein[1], Sebastian Klein[1] and Beat Tödtli[1]

1- Deutsches Elektronen-Synchrotron DESY, Zeuthen
Platanenallee 6 - 15738 Zeuthen, Germany

We calculate the two- and three-loop massive operator matrix elements (OMEs) contributing to the heavy flavor Wilson coefficients of transversity. We obtain the complete result for the two-loop OMEs and compute the first thirteen Mellin moments at three-loop order. As a by-product of the calculation, the moments $N = 1$ to 13 of the complete two-loop and the $T_F$-part of the three-loop transversity anomalous dimension are obtained.



## 1 Framework

The transversity distribution belongs to the three twist-2 parton distribution functions (PDFs), together with those for unpolarized and polarized deep-inelastic scattering. It is a flavor non-singlet, chiral-odd distribution and can be measured in semi-inclusive deep-inelastic scattering (SIDIS) and via the polarized Drell-Yan process. [a] Different experiments perform transversity measurements at the moment, cf. Refs. [2]. Recently, a first phenomenological parameterization has been given for the transversity up- and down-quark distributions in Ref. [3], the moments of which are in qualitative agreement with first lattice calculations [4].

For semi-inclusive deeply inelastic charged lepton-nucleon scattering $lN \to l'h + X$ the scattering cross section is given by

$$\frac{d^3\sigma^{\rm SIDIS}}{dxdydz} = \frac{4\pi\alpha_{\rm em}^2 s}{Q^4}\sum_{a=q,\overline{q}} e_a^2 x\left\{\frac{1}{2}\left[1+(1-y)^2\right]F_a(x,Q^2)D_a(z,Q^2)\right.$$
$$\left. -(1-y)|\mathbf{S}_\perp||\mathbf{S}_{h\perp}|\cos\left(\phi_S+\phi_{S_h}\right)\Delta_T F_a(x,Q^2)\Delta_T D_a(z,Q^2)\right\}, \quad (1)$$

after the $\mathbf{P}_{h\perp}$-integration has been performed, [1]. We consider, for definiteness, only scattering cross sections free of $\mathbf{k}_\perp$- effects to refer to twist–2 quantities. $x$ and $y$ denote the Bjorken variables, $z$ the fragmentation variable, $Q^2 = -q^2$ the space-like 4–momentum transfer, $\alpha_{\rm em}$ the fine structure constant, $e_a$ the quark charge, and $s$ the cms energy squared. $\mathbf{S}_\perp$ and $\mathbf{S}_{h\perp}$ are the transverse spin vectors of the incoming nucleon $N$ and the measured hadron $h$. $F_a(z,Q^2), \Delta_T F_a(z,Q^2)$ and $D_a(z,Q^2), \Delta_T D_a(z,Q^2)$ denote the unpolarized and transversity structure- and fragmentation functions, respectively. The angles $\phi_{S,S_h}$ are measured in the plane transverse to the $\gamma^* N$ axis between the $x$-axis and the respective vector. In process (1) the spin of the *transversely* polarized hadron $h$ has to be measured.

The transversity distribution may also be measured in the transversely polarized Drell-Yan processes. In Mellin space the scattering cross section is given by, [5],

$$\frac{d\Delta_T\sigma^{\rm DY}}{d\phi} = \frac{\alpha_{\rm em}^2}{9s}\cos(2\phi)\Delta_T H(N,M^2)\cdot\Delta_T C_q^{\rm DY}(N,M^2) \quad (2)$$

[a] For a review see Ref. [1].

where $N$ denotes the Mellin variable and $\phi$ is the azimuthal angle of one of the final state leptons $l^{\pm}$ relative to the axis defined by the transverse polarizations.

$$\Delta_T H(N,Q^2) = \sum_q e_q^2 \left[\Delta_T q_1(N,Q^2)\Delta_T \overline{q}_2(N,Q^2) + \Delta_T \overline{q}_1(N,Q^2)\Delta_T q_2(N,Q^2)\right]$$

is a combination of transversity parton distributions for the incoming light (anti-)quarks, and $\Delta_T C_q^{\rm DY}(N,M^2)$ denotes the Wilson coefficient of the Drell-Yan process, with $M^2$ the invariant mass of the produced lepton pair.

Like in the case of unpolarized and polarized deep-inelastic processes transversity receives heavy flavor corrections in higher orders in QCD. These are given by the corresponding heavy flavor Wilson coefficients. As for other non-singlet quantities [6, 7], these corrections start at $O(a_s^2)$, with $a_s = \alpha_s/(4\pi)$. In SIDIS one can tag $Q\bar{Q}$-production in the same way as in the deep-inelastic process, [8]. A measurement is possible in high luminosity experiments. In the Drell-Yan process, on the other hand, heavy flavor contributions emerge inclusively since there the final-state $l^+l^-$-pairs are measured in the first place. The calculation of the heavy quark Wilson coefficients for $Q^2 \gg m^2$ proceeds in the same way as in unpolarized and polarized deep-inelastic scattering [6, 7, 9, 10]

The complete Wilson coefficients for transversity can be decomposed into a light- and a heavy quark contribution

$$C_q^{\rm TR}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) = C_q^{\rm TR,light}\left(x, \frac{Q^2}{\mu^2}\right) + H_q^{\rm TR}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) . \tag{3}$$

As shown in [6], the heavy quark Wilson coefficient for hard processes factorizes into the light quark Wilson coefficients and the massive operator matrix element $A_{qq,Q}^{\rm TR}$ at large enough scales $Q^2 \gg m^2$. We apply this to the heavy flavor Wilson coefficient for transversity $H_q^{\rm TR}$

$$\begin{aligned} H_q^{\rm TR}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) &= C_q^{\rm TR,light}\left(x, \frac{Q^2}{\mu^2}\right) \otimes A_{qq,Q}^{\rm TR}\left(x, \frac{m^2}{\mu^2}\right) \\ &= a_s^2\left[\Delta_T A_{qq,Q}^{(2),\rm NS,TR}(N_f+1) + \Delta_T \hat{C}_q^{(2)}(N_f)\right] + a_s^3\left[\Delta_T A_{qq,Q}^{(3),\rm NS,TR}(N_f+1)\right. \\ &+ \left.\Delta_T A_{qq,Q}^{(2),\rm NS,TR}(N_f+1) \otimes \Delta_T C_q^{(1)}(N_f+1) + \hat{C}_q^{(3)}(N_f)\right] . \end{aligned} \tag{4}$$

The aim of this article is to present a computation of the renormalized two- and three-loop heavy-flavor operator matrix elements contributing to transversity. Details of the calculation are given in Ref. [11]. The operator matrix element $\langle q|O^{\rm NS,TR}|q\rangle$ is given by a two-point Green's function containing a closed loop of a heavy quark $Q$ and external massless quarks $q$. The local operator is given by

$$O_{F,a;\mu\mu_1\ldots\mu_n}^{\rm NS,TR} = i^n \mathbf{S}\left[\overline{\psi}\gamma_5\sigma_{\mu\mu_1}D_{\mu_2}\ldots D_{\mu_n}\frac{\lambda_a}{2}\psi\right] - \text{trace terms} , \tag{5}$$

cf. [12]. Here $\mathbf{S}$ denotes symmetrization of the Lorentz indices, $\sigma_{\mu\nu} = (i/2)\left[\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu\right]$, and $D_\mu$ is the covariant derivative. The Green's function in $D = 4+\varepsilon$ dimensions obeys the following vector decomposition

$$\begin{aligned} \hat{G}_{\mu,q,Q}^{ij,\rm TR,NS} = J_N\langle q|O_{F,a;\mu\mu_1\ldots\mu_n}^{\rm NS,TR}|q\rangle = \delta_{ij}\Big\{&\Delta_\rho\sigma^{\mu\rho}\hat{\hat{A}}_{qq,Q}^{\rm TR,NS}\left(\frac{\hat{m}^2}{\mu^2}, \varepsilon, N\right) + c_1\Delta^\mu \\ &+ c_2 p^\mu + c_3\gamma^\mu \not{p} + c_4 \not{\Delta}\not{p}\Delta^\mu + c_5 \not{\Delta}\not{p}p^\mu\Big\}(\Delta\cdot p)^{N-1} \end{aligned} \tag{6}$$

contracting the OME with a source term $J_N = \Delta^{\mu_1} \ldots \Delta^{\mu_N}$, with $\Delta^2 = 0$, with $p$ the parton momentum. It determines the un-renormalized massive OME

$$\hat{\hat{A}}_{qq,Q}^{\rm TR,NS}\left(\frac{\hat{m}^2}{\mu^2}, \varepsilon, N\right) = \frac{-i\,\delta^{ij}}{4N_c\,(\Delta\cdot p)^{N+1}\,(D-2)}\Biggl\{{\rm Tr}\left[\not\Delta \not p p^\mu \hat{G}^{ij,{\rm TR,NS}}_{\mu,q,Q}\right] - \Delta\cdot p\,{\rm Tr}\left[p^\mu \hat{G}^{ij,{\rm TR,NS}}_{\mu,q,Q}\right] + i\Delta\cdot p\,{\rm Tr}\left[p^\mu \hat{G}^{ij,{\rm TR,NS}}_{\mu,q,Q}\right]\Biggr\}. \quad (7)$$

A total of 129 diagrams contribute, which were generated using `QGRAF` [13]. These were projected onto $\hat{\hat{A}}_{qq,Q}^{TR,NS}$, cf. [9], using codes written in `FORM` [14]. After tensor reduction, the loop integrals are of the tadpole-type, since the single external quark is on-shell and massless. The integrals were then evaluated using `MATAD` [15]. The renormalization of the OMEs is described in Ref. [9].

After mass- and charge renormalization one obtains the massive OMEs in the on-mass-shell scheme, cf. [9],

$$\Delta_T A_{qq,Q}^{(2),{\rm NS},\overline{\rm MS}} = \frac{\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}}}{4}\ln^2\left(\frac{m^2}{\mu^2}\right) + \frac{\hat{\gamma}_{qq}^{(1),{\rm TR}}}{2}\ln\left(\frac{m^2}{\mu^2}\right) + a_{qq,Q}^{(2),{\rm TR}} - \frac{\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}}}{4}\zeta_2\,, \quad (8)$$

$$\begin{aligned}\Delta_T A_{qq,Q}^{(3),{\rm NS},\overline{\rm MS}} = & -\frac{\gamma_{qq}^{(0),{\rm TR}}\beta_{0,Q}}{6}\Bigl(\beta_0 + 2\beta_{0,Q}\Bigr)\ln^3\left(\frac{m^2}{\mu^2}\right) + \frac{1}{4}\Biggl\{2\gamma_{qq}^{(1),{\rm TR}}\beta_{0,Q} \\ & -2\hat{\gamma}_{qq}^{(1),{\rm TR}}\Bigl(\beta_0 + \beta_{0,Q}\Bigr) + \beta_{1,Q}\gamma_{qq}^{(0),{\rm TR}}\Biggr\}\ln^2\left(\frac{m^2}{\mu^2}\right) + \frac{1}{2}\Biggl\{\hat{\gamma}_{qq}^{(2),{\rm TR}} \\ & -\Bigl(4a_{qq,Q}^{(2),{\rm TR}} - \zeta_2\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}}\Bigr)(\beta_0 + \beta_{0,Q}) + \gamma_{qq}^{(0),{\rm TR}}\beta_{1,Q}^{(1)}\Biggr\}\ln\left(\frac{m^2}{\mu^2}\right) \\ & +4\overline{a}_{qq,Q}^{(2),{\rm TR}}(\beta_0 + \beta_{0,Q}) - \gamma_{qq}^{(0)}\beta_{1,Q}^{(2)} - \frac{\gamma_{qq}^{(0),{\rm TR}}\beta_0\beta_{0,Q}\zeta_3}{6} - \frac{\gamma_{qq}^{(1),{\rm TR}}\beta_{0,Q}\zeta_2}{4} \\ & +2\delta m_1^{(1)}\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}} + \delta m_1^{(0)}\hat{\gamma}_{qq}^{(1),{\rm TR}} + 2\delta m_1^{(-1)}a_{qq,Q}^{(2),{\rm TR}} + a_{qq,Q}^{(3),{\rm TR}}\,, \quad (9)\end{aligned}$$

at 2– and 3–loops. Here, $\zeta_k$ denotes the Riemann $\zeta$-function at $\gamma_{qq}^{(l),{\rm TR}}$ are the transversity anomalous dimensions for $l = 0, 1, 2$ in LO [16], NLO [5,17], and NNLO [18], with $\hat{f}(N_f) = f(N_f + 1) - f(N_f)$. For the other quantities we refer to [9]. The new terms being calculated are $a_{qq,Q}^{(2),{\rm TR}}(N)$, $\overline{a}_{qq,Q}^{(2),{\rm TR}}(N)$ and $a_{qq,Q}^{(3),{\rm TR}}(N)$, and for the higher values of $N$, $\hat{\gamma}_{qq}^{(2),{\rm TR}}(N)$.

## 2 Results

### 2.1 Massive Operator Matrix Elements

At $O(a_s^2)$ the massive operator matrix elements for transversity $\Delta_T A_{qq,Q}^{(2),{\rm NS},\overline{\rm MS}}$ are obtained for general values of $N$, cf. Eq. (8). The un-renormalized OME is computed to $O(\varepsilon)$ to also extract the functions $\overline{a}_{qq,Q}^{{\rm TR},(2)}(N)$. The new terms at 2–loops are $a_{qq,Q}^{{\rm TR},(2)}$ and $\overline{a}_{qq,Q}^{{\rm TR},(2)}$, cf.

Eqs. (8, 9):

$$a_{qq,Q}^{\mathrm{TR},(2)}(N) = C_F T_F \Biggl\{ -\frac{8}{3} S_3 + \frac{40}{9} S_2 - \left[\frac{224}{27} + \frac{8}{3}\zeta_2\right] S_1 + 2\zeta_2 + \frac{\left(24 + 73\,N + 73\,N^2\right)}{18N\,(1+N)} \Biggr\} \tag{10}$$

$$\begin{aligned}\overline{a}_{qq,Q}^{\mathrm{TR},(2)}(N) = C_F T_F \Biggl\{ & -\left[\frac{656}{81} + \frac{20}{9}\zeta_2 + \frac{8}{9}\zeta_3\right] S_1 + \left[\frac{112}{27} + \frac{4}{3}\zeta_2\right] S_2 - \frac{20}{9} S_3 \\ & + \frac{4}{3} S_4 + \frac{1}{6}\zeta_2 + \frac{2}{3}\zeta_3 + \frac{\left(-144 - 48\,N + 757\,N^2 + 1034\,N^3 + 517\,N^4\right)}{216 N^2 \,(1+N)^2} \Biggr\} ,\end{aligned} \tag{11}$$

with $S_k \equiv S_k(N)$ the single harmonic sums.

At $O(a_s^3)$ the moments $N = 1$ to 13 were computed for the massive OMEs, as e.g.

$$\begin{aligned}\Delta_T\ A_{qq,Q}^{(3),\mathrm{NS},\overline{\mathrm{MS}}}(13) \quad = \quad & C_F T_F \Biggl\{ \Bigl(\frac{1751446}{110565} C_A - \frac{7005784}{1216215} T_F (N_f + 2)\Bigr) \ln^3\Bigl(\frac{m^2}{\mu^2}\Bigr) \\ & + \Bigl(-\frac{20032048197492631}{2193567563187000} C_F - \frac{137401473299}{8027019000} C_A - \frac{93611152819}{3652293645} T_F\Bigr) \ln^2\Bigl(\frac{m^2}{\mu^2}\Bigr) \\ & + \Bigl[\Bigl(\frac{170583232732904244998 3}{26349133569002244 0000} + \frac{7005784}{45045}\zeta_3\Bigr) C_F + \Bigl(\frac{338545448824819123 7}{6580702689561000 0} \\ & - \frac{7005784}{45045}\zeta_3\Bigr) C_A - \frac{4581147910764137 71}{658070268956100 0} N_f T_F - \frac{217179304}{3648645} T_F\Bigr] \ln\Bigl(\frac{m^2}{\mu^2}\Bigr) \\ & + \Bigl(-\frac{7005784}{135135} B_4 + \frac{3502892}{15015}\zeta_4 - \frac{81735983092}{243486243}\zeta_3 \\ & + \frac{5537627829952273383742505249 3}{1220808056519011969008000 00}\Bigr) C_F + \Bigl(\frac{3502892}{135135} B_4 - \frac{3502892}{15015}\zeta_4 \\ & + \frac{4061479439}{12162150}\zeta_3 - \frac{34868969747438825567756 47}{12935029206601101600 000}\Bigr) C_A \\ & + \Bigl(-\frac{2799227526321603558606 97}{35571330318153029400 00} + \frac{56046272}{1216215}\zeta_3\Bigr) T_F N_f \\ & + \Bigl(\frac{2915265503027600701553 03}{71142660636306058800 00} - \frac{14011568}{173745}\zeta_3\Bigr) T_F \Biggr\} ,\end{aligned} \tag{12}$$

where

$$B_4 = -4\zeta_2 \ln^2(2) + \frac{2}{3}\ln^4(2) - \frac{13}{2}\zeta_4 + 16\mathrm{Li}_4\left(\frac{1}{2}\right) .$$

Like for the massive OMEs in case of unpolarized deep-inelastic scattering, the structure of $\Delta_T\ A_{qq,Q}^{(3),\mathrm{NS},\overline{\mathrm{MS}}}(N)$ is widely known for general values of $N$, except the contributions due to the finite part $a_{qq,Q}^{(3),NS}$ and the 3-loop anomalous dimension $\hat{\gamma}_{qq}^{(2),\mathrm{TR}}(N)$. One notices the cancellation of all $\zeta_2$ terms in $\Delta_T\ A_{qq,Q}^{(3),\mathrm{NS},\overline{\mathrm{MS}}}(N)$ after renormalization.

### 2.2 Anomalous Dimensions

The transversity anomalous dimension is given by

$$\gamma_{qq}^{\mathrm{TR}}(N, a_s) = \sum_{i=1}^{\infty} a_s^i \gamma_{qq}^{(i),\mathrm{TR}}(N)\,. \tag{13}$$

From Eq. (9) one may determine the complete 2-loop anomalous dimension [5, 17] and the $T_F$-part of the 3-loop anomalous dimension [18]. We agree with the results given in [5,17] and confirm the $T_F$-contributions for the moments $N = 1$ to 8 given in Refs. [18]. Furthermore, we obtain $\hat{\gamma}_{qq}^{(3),\mathrm{TR}} = \gamma_{qq}^{(3),\mathrm{TR}}(N_f + 1) - \gamma_{qq}^{(3),\mathrm{TR}}(N_f)$ newly for $N = 9$ to 13, as e.g.

$$\hat{\gamma}_{qq}^{(3),\mathrm{TR}}(N = 13) = -\,C_F T_F \left[\frac{36713319015407141570017}{131745667845011220000}C_F - \frac{14011568}{45045}(C_F - C_A)\zeta_3 \right.$$
$$\left. + \frac{66409807459266571}{3290351344780500}T_F(1 + 2N_f) + \frac{6571493644375020121}{65807026895610000}C_A\right].$$

### 2.3 A Remark on the Soffer Bound

If the Soffer inequality [19]

$$|\Delta_T f(x, Q^2)| \le \frac{1}{2}\left[f(x, Q^2) + \Delta f(x, Q^2)\right] \tag{14}$$

holds for the non-perturbative PDFs in Eq. (14) one may check its generalization from $f_i \to F_i$ for the corresponding structure functions. This includes the non-singlet evolution operator (Eq. (6), Ref. [20]) and the heavy flavor Wilson coefficient. At perturbative scales, it holds for the evolution operator [11], generalizing a result from [5] for the moments $N = 1$ to 13 at 3–loops. For the heavy quark Wilson coefficients in SIDIS we only know the massive OMEs so far. As shown in Ref. [11], a final conclusion can only be drawn knowing the yet undetermined massless Wilson coefficients. Here the difference $[A_{qq,Q}^{\mathrm{V}} - A_{qq,Q}^{\mathrm{TR}}](x)$ of the massive OMEs, shows a sign change to negative values for $Q^2/m^2$ in the physical range. For large scales $Q^2/m^2 \gg 1$ positive values are obtained.

## References


[1] V. Barone, A. Drago and P. G. Ratcliffe, Phys. Rept. **359** (2002) 1 [arXiv:hep-ph/0104283].

[2] A. Airapetian *et al.* [HERMES Collaboration], Phys. Rev. Lett. **94** (2005) 012002 [arXiv:hep-ex/0408013];
A. Airapetian *et al.* [HERMES Collaboration], JHEP **0806** (2008) 017 [arXiv:0803.2367 [hep-ex]];
M. Alekseev *et al.* [COMPASS Collaboration], Phys. Lett. B **673** (2009) 127 [arXiv:0802.2160 [hep-ex]]; V. Y. Alexakhin *et al.* [COMPASS Collaboration], Phys. Rev. Lett. **94** (2005) 202002 [arXiv:hep-ex/0503002];
COMPASS collaboration, private communication.
M. F. Lutz, B. Pire, O. Scholten and R. Timmermans et al., [The PANDA Collaboration], Physics Performance Report for PANDA: Strong Interaction Studies with Antiprotons, arXiv:0903.3905 [hep-ex];
A. Afanasev *et al.*, arXiv:hep-ph/0703288.

[3] M. Anselmino *et al.*, arXiv:0807.0173 [hep-ph].

[4] S. Aoki, M. Doui, T. Hatsuda and Y. Kuramashi, Phys. Rev. D **56** (1997) 433 [arXiv:hep-lat/9608115]; M. Göckeler *et al.*, Nucl. Phys. Proc. Suppl. **53** (1997) 315 [arXiv:hep-lat/9609039]; A. Ali Khan *et al.*, Nucl. Phys. Proc. Suppl. **140** (2005) 408 [arXiv:hep-lat/0409161]; D. Dolgov *et al.* [LHPC collaboration and TXL Collaboration], Phys. Rev. D **66** (2002) 034506 [arXiv:hep-lat/0201021]; M. Diehl *et al.* [QCDSF Collaboration and UKQCD Collaboration], arXiv:hep-ph/0511032; M. Göckeler *et al.* [QCDSF Collaboration and UKQCD Collaboration], Phys. Rev. Lett. **98** (2007) 222001 [arXiv:hep-lat/0612032]; D. Renner, private communication.

[5] W. Vogelsang, Phys. Rev. D **57** (1998) 1886 [arXiv:hep-ph/9706511].

[6] M. Buza, Y. Matiounine, J. Smith, R. Migneron and W. L. van Neerven, Nucl. Phys. B **472** (1996) 611 [arXiv:hep-ph/9601302];
M. Buza, Y. Matiounine, J. Smith and W. L. van Neerven, Nucl. Phys. B **485** (1997) 420 [arXiv:hep-ph/9608342].

[7] I. Bierenbaum, J. Blümlein and S. Klein, Nucl. Phys. B **780** (2007) 40 [arXiv:hep-ph/0703285];
I. Bierenbaum, J. Blümlein, S. Klein and C. Schneider, Nucl. Phys. B **803** (2008) 1 [arXiv:0803.0273 [hep-ph]].

[8] S. Alekhin, J. Blumlein, S. Klein and S. Moch, arXiv:0908.2766 [hep-ph].

[9] I. Bierenbaum, J. Blümlein and S. Klein, Nucl. Phys. B **820** (2009) 417 [arXiv:0904.3563 [hep-ph]]; arXiv:0907.2615 [hep-ph].

[10] M. Buza, Y. Matiounine, J. Smith and W. L. van Neerven, Eur. Phys. J. C **1** (1998) 301 [arXiv:hep-ph/9612398];
J. Blümlein, A. De Freitas, W. L. van Neerven and S. Klein, Nucl. Phys. B **755** (2006) 272 [arXiv:hep-ph/0608024];
I. Bierenbaum, J. Blümlein and S. Klein, Phys. Lett. B **672** (2009) 401 [arXiv:0901.0669 [hep-ph]]; arXiv:0706.2738 [hep-ph].

[11] J. Blümlein, S. Klein, and B. Tödtli, DESY 09–60, `arXiv: 0909.1547`.

[12] B. Geyer, D. Robaschik and E. Wieczorek, Fortsch. Phys. **27** (1979) 75.

[13] P. Nogueira, J. Comput. Phys. **105** (1993) 279.

[14] J. A. M. Vermaseren, arXiv:math-ph/0010025.

[15] M. Steinhauser, Comput. Phys. Commun. **134** (2001) 335 [arXiv:hep-ph/0009029].

[16] F. Baldracchini, N. S. Craigie, V. Roberto and M. Socolovsky, Fortsch. Phys. **30** (1981) 505 [Fortsch. Phys. **29** (1981) 505];
M. A. Shifman and M. I. Vysotsky, Nucl. Phys. B **186** (1981) 475;
A. P. Bukhvostov, G. V. Frolov, L. N. Lipatov and E. A. Kuraev, Nucl. Phys. B **258** (1985) 601;
J. Blümlein, Eur. Phys. J. C **20** (2001) 683 [arXiv:hep-ph/0104099];
A. Mukherjee and D. Chakrabarti, Phys. Lett. B **506** (2001) 283 [arXiv:hep-ph/0102003].

[17] A. Hayashigaki, Y. Kanazawa and Y. Koike, Phys. Rev. D **56** (1997) 7350 [arXiv:hep-ph/9707208];
S. Kumano and M. Miyama, Phys. Rev. D **56** (1997) 2504 [arXiv:hep-ph/9706420];

[18] J. A. Gracey, Nucl. Phys. B **662** (2003) 247 [arXiv:hep-ph/0304113]; Nucl. Phys. B **667** (2003) 242 [arXiv:hep-ph/0306163]. JHEP **0610** (2006) 040 [arXiv:hep-ph/0609231]; Phys. Lett. B **643** (2006) 374 [arXiv:hep-ph/0611071].

[19] J. Soffer, Phys. Rev. Lett. **74** (1995) 1292 [arXiv:hep-ph/9409254].

[20] J. Blümlein, H. Böttcher and A. Guffanti, Nucl. Phys. B **774** (2007) 182 [arXiv:hep-ph/0607200].

[21] Slides: `http://indico.cern.ch/contributionDisplay.py?contribId=130&sessionId=4&confId=53294`